A New Methodology for Measuring Actual to Expected Performance

By Daniel Bauer and Jochen Russ.

Introduction
The quality of life expectancy estimates (LEs) is one of the most important considerations for an investor in Life Settlements. The predominant metric for assessing this quality is an ex-post comparison of the Actual number of deaths to the predicted or Expected number of deaths, the “A/E ratio”. If the actual number is close to the expected number, then the ratio of actual to expected deaths obviously will be close to 100%—which typically is interpreted as an indication of a high quality of medical underwriting.

We explain in this article why such an interpretation can be deceiving at certain durations. More precisely, we show that the A/E ratio can be “biased” in that it is automatically pulled towards 100% over time, rendering it potentially misleading for older portfolios. As an alternative, we propose a new methodology for measuring actual to expected performance via the difference in life expectancies (DLE) and the difference in curtate life expectancies (DCLE). We provide illustrative calculations documenting the superiority of this approach, and we use it to analyze the Fasano Associates database. The results show a relatively small difference between actual and expected curtate life expectancies in the Fasano Associates data.

Weakness of current A/E methodology
While A/E ratios are often a suitable measure to assess the quality of life expectancy estimates, they can be misleading when used to evaluate life expectancy estimates issued a long time ago. Ultimately everyone dies, at which point the A/E ratio becomes 100%. Everyone could have died well before or well after their LE, but if you wait long enough to perform your A/E analysis, ultimately the A/E ratio will become 100%.

Figure 1 (below) illustrates the shortcoming of current A/E methodology for a simple hypothetical example: The top panel shows the development of the actual number of deaths (red curve), the expected number of deaths for a hypothetical aggressive LE provider with LEs that were too short by two years (blue curve) and the expected number of deaths for a hypothetical conservative LE provider with LEs that were too long by two years (green curve). The bottom panel shows the corresponding evolution of A/E ratios for the aggressive LE provider (blue curve) and for the conservative LE provider (green curve).

We observe that after about 10 years, for both LE providers, the expected number of deaths starts approaching the actual number of deaths. Hence, as illustrated in the bottom panel, A/E ratios approach 100%, even if one provider was significantly too short and the other was significantly too long.

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1 This article is based on a talk the authors gave at the Fasano 9th Annual Life Settlement & Longevity Conference. The slides from the talk are available at www.ifas-ulm.de/downloads/DCLE.pdf. The talk and this article are based on a joint research project by the authors and Nan Zhu, Illinois State University.

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Thus, we can conclude that A/E ratios are “artificially” pulled towards 100% if a significant portion of a portfolio of lives was underwritten a rather long time ago.

Figure 1: Top panel: Development of actual number of deaths (red curve), expected number of deaths for an aggressive LE provider (blue curve), and expected number of deaths for a conservative LE provider (green curve) in a simple, hypothetical example. Bottom panel: The corresponding evolution of the A/E ratios for the aggressive LE provider (blue curve) and the conservative LE provider (green curve).
A new methodology

A better metric would be the difference between the projected and the realized life expectancy – or DLE: Assume all individuals within a certain portfolio have died. Then, for each of them, one can calculate the difference between the actual time lived and the projected life expectancy. Clearly, for some this difference will be positive, and for some it will be negative by the sheer randomness of the time of death. However, if the average of these differences is positive, that would be a strong indication that the projected expectancies were too short—especially if the portfolio is large—whereas a negative average would indicate that projected life expectancies were too long.

However, typically not all individuals will have died when one wants to assess the quality of underwriting. Thus, we propose a new methodology that is based on the difference between the actual number of months lived until today and the expected number of months lived until today. In the simple example from Figure 1, the deviation between actual and expected mortality would not be measured by the ratio of the respective curves but rather by the area between the respective curves. At any point in time the area between the green and the red curve is a measure for the amount of time people have lived shorter than was originally projected by the conservative LE provider. Similarly, the area between the blue and the red curve is a measure for the amount of time people have lived longer than was originally projected by the aggressive LE provider. We call this difference between actual and expected number of months lived to date (divided by the number of lives in the considered portfolio) the Difference in Curtate Life Expectancy or DCLE.

As time goes by, the respective areas become larger until finally, the whole difference between projected and actual life expectancy will have accrued — i.e., the Difference in Curtate Life Expectancy (DCLE) approaches the Difference in Life Expectancy (DLE). In the simple hypothetical example, we have assumed that LE provider 1 (blue) gave life expectancy estimates that were systematically too short by 2 years whereas LE provider 2 (green) gave life expectancy estimates that were systematically too long by 2 years — i.e., we assumed DLEs of +2 and -2 years, respectively. Figure 2 (below) shows how the DCLE, i.e. the difference between the actual number of months lived until today and the expected number of months lived until today evolves over time until finally it correctly identifies that both LE providers were off by 24 months in opposite directions.

Figure 2: Development of DCLE for an aggressive LE provider (blue curve) and a conservative LE provider (green curve) in a simple, hypothetical example.
Results based on actual Fasano Associates data

We applied the new methodology explained above to the entire Fasano Associates database, which consists of more than 50,000 unique lives. We performed a variety of analyses for the whole portfolio as well as for certain segments of the portfolio, e.g. segmented by time of underwriting, by mortality multiplier, etc.

Table 1 (below) summarizes the DCLE results for the Fasano Associates database:

<table>
<thead>
<tr>
<th>Sorted by Date of Underwriting:</th>
<th>3/1/2001 to 12/30/04</th>
<th>1/1/05 to 3/21/07</th>
<th>3/22/07 to 4/30/08</th>
<th>After 4/30/08</th>
<th>All LEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.86 months</td>
<td>0.69 months</td>
<td>-0.04 months</td>
<td>-0.27 months</td>
<td>0.76 months</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sorted by Mortality Multiplier or Rating (MR):</th>
<th>MR &lt; 120%</th>
<th>MR between 120% and 250%</th>
<th>MR &gt; 250%</th>
<th>All LEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.57 months</td>
<td>1.37 months</td>
<td>-0.83 months</td>
<td>0.76 months</td>
<td></td>
</tr>
</tbody>
</table>

All DCLE values were reasonably close to zero. In particular, we found DCLE values of 0.76 months for the entire Fasano portfolio, of just below 5 months for cases underwritten before 2004, and of less than 1 month for cases underwritten thereafter.

It is important to note, however, that DCLE measures the difference between actual and expected number of months lived between the time of underwriting and today. In other words, it is a cumulative deviation that keeps accumulating until all individuals in the portfolio have died. Therefore, results for cases underwritten a long time ago have more statistical credibility than recently underwritten cases. In that regard, it is the perfect compliment to A/E analyses that become less meaningful as more time passes from the date of underwriting.

Summary

In this article, we show that current approaches of measuring A/E ratios can be problematic, as A/E ratios are artificially pulled towards 100% over time and therefore may overestimate the quality of LE providers if a significant portion of their LEs has been underwritten a rather long time ago. We therefore propose a new approach based on the Difference in Life Expectancy (DLE) and Difference in Curtate Life Expectancy (DCLE). The DCLE is calculated as the average difference between the actual number of months lived until today and expected number of months lived until today. This metric has no structural statistical bias, and converges towards the DLE over time. That is, as more and more information about the difference between projected LEs and actual mortality becomes available, the deviation is accumulated and the difference is evaluated more and more correctly. An analysis of Fasano Associates’ data shows a rather small difference between actual and expected (curtate) life expectancies.